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ЗАСТОСУВАННЯ МЕТОДУ ДЕФОРМОВАНИХ КООРДИНАТ ДО ВИЗНАЧЕННЯ ХВИЛЬОВИХ ХАРАКТЕРИСТИК СКІНЧЕННОЇ АМПЛІТУДИ НА ГЛИБОКОВОДДІ І МІЛКОВОДДІ

О.М. Воробйова
к.т.н., доцент,
зав. кафедрою «Автоматизації та ком'ютерно-інтегрованих технологій»
vorobyovaem14@gmail.com

Державний університет інтелектуальних технологій, Одеса, Україна

К.Ю. Федорова
к.т.н., доцент кафедри «Морське та цивільне будівництво та архітектура»
fedorovaky@yahoo.com
О.К. Рожко
старший викладач, аспірант кафедри «Машинознавство й інженерна механіка»
ooonlineshop@gmail.com

Одеський національний морський університет, Одеса, Україна

Анотація. *Розглянуті особливості визначення характеристик хвилювання при використанні метода деформованих координат. Наведено порівняння профілів хвиль, які розраховані для мілководної акваторії за наближеною теорією, що запропонована, за теорією Стокса та за кноїдальною теорією. Вивчення хвильових рухів на вільній поверхні вод морів та океанів розвивається за чотирма напрямками: гідродинамічному, енергетичному, статистичному та спектральному на основі використання теоретичних та експериментальних методів. Сутність гідродинамічної теорії хвиль полягає у математичному вивченні хвильових рухів ідеальної рідини із вільною поверхнею. Ця теорія дозволяє вірно оцінити внутрішню динамічну структуру хвильового руху, зв'язки між окремими елементами хвиль. Відповідь на одне з основних питань у теорії морських хвиль: чому ледве помітна вітрова бриж під дією сильного вітру виростає на океанських просторах в гігантські хвилі і який механізм їх гасіння дозволяє отримати енергетична теорія?*

Відмінною властивістю морських хвиль є складність і відома хаотичність структури схвильованої поверхні. Вивчення цього питання ведеться з двох точок зору. З одного боку, досліджуються статистичні закони розподілу безпосередньо спостерігаємих елементів хвиль-висот, періодів, довжин; з іншого боку, вивчається внутрішня спектральна структура поверхні, її енергетичний спектр.

Ключові слова: *глибока вода, мілководдя, нелінійна теорія хвиль, метод деформованих координат, профіль хвилі.*

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APPLICATION OF THE DEFORMABLE COORDINATE METHOD TO THE DETERMINATION OF FINITE AMPLITUDE'S WAVE CHARACTERISTICS IN DEEP WATER AND SHALLOW WATER

O. Vorobyova
PhD, associate professor
of the Department of «Automation and computer-integrated technologies»
vorobyovaem14@gmail.com

State University of Intellectual Technologies, Odesa, Ukraine

К. Fedorova
PhD, associate professor
of the Department of «Marine and civil construction and architecture»
fedorovaky@yahoo.com
O. Rozhko
assistant professor, PhD Student
of the Department of «Machine science and mechanical engineering»
ooonlineshop@gmail.com

Odesa National Maritime University, Odesa, Ukraine

Abstract. *Specifics of defining characteristics for finite amplitude waves with the help of deformed coordinates method for deep water and shallowness are shown. Comparison of shallow water wave profiles for proposed approximate theory, Stokes and cnoidal theories is introduced. The study of wave movements on the free surface of the waters of seas and oceans develops in four directions: hydrodynamic, energetic, statistical and spectral based on the use of theoretical and experimental methods. The essence of the hydrodynamic theory of waves is the mathematical study of the wave motions of an ideal fluid with a free surface.*

This theory makes it possible to accurately assess the internal dynamic structure of wave motion, the connections between individual elements of waves. The answer to one of the main questions in the theory of sea waves: why does a barely noticeable wind ripple under the action of a strong wind grow in the ocean spaces into giant waves and what mechanism of their extinguishing allows the energy theory to be obtained? A distinctive feature of sea waves is the complexity and known chaos of the structure of the agitated surface.

The study of this issue is conducted from two points of view. On the one hand, the statistical laws of the distribution of the directly observable elements of waves-heights, periods, and lengths are studied; on the other hand, the internal spectral structure of the surface, its energy spectrum, is studied.

Keywords: *deep water, shallow water, non-linear wave theory, deformed coordinates method, wave profile.*

Introduction. Safe operation of the Ukrainian maritime industry is impossible without knowledge of wave characteristics. Both extreme data intended for the survival mode of objects and operational data for the established operating mode are required.

The depths in most of the Ukrainian sector of the Black Sea water area do not exceed 100 m, and the average depth of the sea is more than 1200 m [1]. Since under these conditions the existence of both short and sufficiently long waves is possible, the wave parameters must be determined for both limited and infinite depth.

Waves of extreme steepness outside the surf zone occur quite rare; expressions defining wave characteristics finite amplitude are complex and cumbersome. Therefore, for engineering calculations, the results of the linear theory of wave motion are mainly used.

Consequently, the derivation and verification of a compact approximate expression for the profile of progressive waves of finite amplitude in a liquid of arbitrary depth seems to be a very urgent problem.

Analysis of major achievements and literature. The characteristics of waves of finite amplitude have been studied by domestic and numerous foreign authors for more than half a century. The main results were obtained within the framework of the potential theory.

Among the large number of existing nonlinear theories of waves in a limited water area, the most widespread are the Stokes theory and the cnoidal theory.

When using Stokes theory in engineering applications, as a rule, one is limited to five terms in the series expansion of the wave surface equation in slope (for example). In [2], as an example, a comparison of wave characteristics obtained from third- and fifth-order theories is given. There are [2] expansions up to the eleventh order.

It should be noted [2] the use of Chapeler's theory, where the expansion coefficients are determined numerically by the least squares method (by minimizing errors in the boundary conditions on the free surface), as well as [2] the use of the so-called new wave theory, which allows one to obtain a linear approximation to the most probable form maximum wave in a storm.

For very shallow depths (less than a tenth of a wavelength), it is usually recommended to use the cnoidal wave theory. It is known [6] that the expansion is up to the ninth order for a solitary wave and up to the fifth order for cnoidal waves. In [7], the existence of expansions of the fourteenth, seventeenth and twenty-seventh orders for solitary waves is indicated.

The works [8, 9] provide approximate expressions for the velocity potentials of free progressive waves of finite amplitude in shallow and deep water, respectively; an assessment of the quality of the resulting solution is given below.

The applicability of various theories of wave formation can be assessed by the values of the Ursell number $N_{URS} = \frac{H\lambda^2}{h^3}$ [5]: when $N_{URS} < 10$ – Stokes theory, when $N_{URS} > 26$ – cnoidal theory. In between, both theories are applicable except when, under the same conditions, the linear theory can be used. According to other sources, the cnoidal theory should be applied when $N_{URS} > 40$ [6].

The use of nonlinear theory is appropriate for wind waves of extreme steepness in deep water, for long waves in significant shallow water and in the zone of wave destruction.

According to data [1], the period of wind waves in the Black Sea, as a rule, does not exceed 9 s. The length of such a wave in deep water conditions is 126 m. The height of the waves does not exceed 6 m (except for the largest storm waves). Swell waves in the Black Sea are long-period – 13-15 s. The lengths of such waves in deep water are respectively 264-351 m, heights are up to 11 m.

In the shallow water zone (depth range – from half the wavelength to the critical depth), the wave profile changes. The tops become sharper, the soles become flatter). Large waves decrease, small ones lengthen and increase in height [6]. However, the average periods and distributions of periods of wind waves during the transition from deep water to shallow water practically do not change (statistical data) [5].

Wave destruction begins at a critical depth equal to twice the wave height. The lower the initial steepness of deep water waves, the more their height increases and their destruction begins at a smaller relative depth. For example, a swell wave with a period of 13 s and a steepness of 1/50 begins to collapse at a depth of 10,6 m, and with a steepness of 1/30 – at a depth of 22,3 m; with a wave period of 15 s, these depths are 14 m and 27,6 m, respectively.

The purpose of the research is to determine the wave profile of finite amplitude in deep and shallow water conditions using the deformable coordinate method. Comparison of wave profiles calculated for shallow water areas using the proposed approximate theory, Stokes theory and cnoidal theory.

Formulation of the problem. We will assume that the fluid is ideal, heavy, incompressible, and its motion is potential. To describe the wave motion of a fluid in a fluid of arbitrary depth h, we use fixed and moving coordinate systems. As is customary in hydrodynamic problems, the origin of the fixed Cartesian coordinate system $Oxyz$ is compatible with the free surface, x_1 axis is directed to the direction of wave propagation, the z_1 axis is vertically upward. The moving Cartesian coordinate system $Oxyz$ at $t = 0$ coincides with the fixed one and moves uniformly in the direction of wave travel with speed c. The velocity potential $\varphi(x_1, z_1, t)$ and wave profile are periodic with a period equal to the wavelength λ.

Let us proceed, as was done in [8], to the dimensionless characteristics

$$\tilde{x} = k(x_1 - c \cdot t);\ \tilde{z} = k \cdot z_1;\ N = kh;\ k = 2 \cdot \frac{\pi}{\lambda};$$
$$\phi(x_1 - c \cdot t.z_1) = \frac{c}{k}\tilde{\Phi}(x, z);\ z_в(x_1 - c \cdot t) = \frac{1}{k}\tilde{\xi}_в(x), \tag{1}$$

here $z_в = z_в(x_1, t)$ – is the equation of the free surface of an excited fluid.

In the following presentation, we will omit the sign «~» over dimensionless coordinates and functions. Note also that the function $\xi_{в}(x) - 2\pi$ – is periodic.

The domain of determination of the potential $\Phi(x,z)$ is respectively:

– in deep water – lower half-plane $z \le 0$;

– in shallow water of depth *h* – a stripe $\{-\infty < x < \infty, -H \le z \le 0\}$.

This area is a physical space in which the kinematic and hydrodynamic characteristics of the wave motion under study are described.

Following the procedure outlined in [8; 9], perform the change of variables:
for unlimited depth

$$x = \xi + F(\xi,\eta),\ z = \eta; \tag{2}$$

for shallow water

$$x = \xi + K(\xi,\eta),\ z = \eta. \tag{3}$$

The functions $F(\xi,\eta)$ and $K(\xi,\eta)$, which deform space, are defined according to [8; 9], their specific form is given below. Further, instead of the normalized physical space (x, z), we will use the conditional deformed space (ξ, η).

In the space (ξ, η) for the potential $\Phi(x,z)$ the following expressions are obtained:

for deep water [9]

$$\Phi(\xi,\eta) = A\exp(\omega\eta)\sin\omega\xi, \tag{4}$$

where

$$\omega = \frac{g}{kc^2} = \frac{1}{2\pi}\frac{g\lambda}{c^2};$$

for shallow water [8]

$$\Phi(\xi,\eta) = Bch\,\alpha(\eta + H)\sin\alpha\xi - \\ - \frac{1}{2}B^2\alpha^3\frac{ch\,2\alpha(\eta + H)\sin 2\alpha\xi}{\omega\alpha\,sh\,2\alpha H - 2\alpha^2 ch\,2\alpha H} - \frac{1}{8}B^2\alpha\sin 2\alpha\xi, \tag{5}$$

where α – is the wave number in the deformed space (ξ,η), associated with the ω dispersion relation

$$\alpha = \omega\,th\,\alpha H. \tag{6}$$

The constants A in formula (4) and B in formula (5), given in [9] and [8] respectively without derivation, are defined below.

Let us denote h_+ the height of the wave crest above the undisturbed free surface, and h_- – the distance of the base from the undisturbed free surface. Then in the normalized physical coordinate system (x, z) the value $kr_в = \frac{k}{2}(h_+ + h_-)$ represents the half-height of the wave.

Research materials. The equation of the wave profile in the coordinate system (x, z), corresponding to the nonlinear boundary condition on the free surface, has the form

$$\xi_в(x) = \frac{1}{\omega}\frac{\partial}{\partial x}\Phi\left[x, z = \xi_в(x)\right] - \\ -\frac{1}{2}\frac{1}{\omega}\left\{\left[\frac{\partial}{\partial x}\Phi\left[x, z = \xi_в(x)\right]\right]^2 + \left[\frac{\partial}{\partial z}\Phi\left[x, z = \xi_в(x)\right]\right]^2\right\}. \tag{7}$$

Expanding the derivatives of the potential $\Phi(x, z)$ in powers of $\xi_в(x) = 0(\varepsilon)$ and preserving terms up to the second order of smallness inclusive, we find

$$\xi_в(x) = \frac{1}{\omega}\left[\frac{\partial}{\partial x}\Phi(x,0) + \xi_в(x)\frac{\partial^2}{\partial x \partial z}\Phi(x,0)\right] - \\ \frac{1}{2}\frac{1}{\omega}\left\{\left[\frac{\partial}{\partial x}\Phi(x,0)\right]^2 + \left[\frac{\partial}{\partial z}\Phi(x,0)\right]^2\right\} - , \tag{8}$$

when follows

$$\xi_в(x) = \frac{1}{\omega}\left[\frac{\partial}{\partial x}\Phi(x,0) + \frac{1}{\omega}\cdot\frac{\partial}{\partial x}\Phi(x,0)\cdot\frac{\partial^2}{\partial x \partial z}\Phi(x,0)\right] - \\ -\frac{1}{2}\cdot\frac{1}{\omega}\left\{\left[\frac{\partial}{\partial x}\Phi(x,0)\right]^2 + \left[\frac{\partial}{\partial z}\Phi(x,0)\right]^2\right\}. \tag{9}$$

Considering the dependencies between the derivatives of potentials written in normalized physical space and deformed space and still preserving terms up to the second order of smallness inclusive, we write in the variables (ξ, η) of the deformed space

$$\xi_в(\xi)=\frac{1}{\omega}\left[\frac{\partial}{\partial\xi}\Phi(\xi,0)-\frac{\partial}{\partial\xi}\Phi(\xi,0)\frac{\partial}{\partial\xi}\Phi(\xi,0)\right]+\frac{1}{\omega}\cdot\frac{1}{\omega}\cdot\frac{\partial}{\partial\xi}\Phi(\xi,0)\times$$
$$\times\frac{\partial^2}{\partial\xi\partial\eta}\Phi(\xi,0)-\frac{1}{2}\cdot\frac{1}{\omega}\left\{\left[\frac{\partial}{\partial\xi}\Phi(\xi,0)\right]^2+\left[\frac{\partial}{\partial\eta}\Phi(\xi,0)\right]^2\right\}. \quad (10)$$

The function $F(\xi,\eta)$ is written [9] in the form

$$F(\xi,\eta)=-\frac{1}{2}\Phi(\xi,\eta)=-\frac{1}{2}A\exp(\omega,\eta)\sin\omega\xi, \quad (11)$$

then

$$\xi_в(\xi)=\frac{1}{\omega}\left\{\frac{\partial}{\partial\xi}\Phi(\xi,0)+\frac{1}{\omega}\cdot\frac{\partial}{\partial\xi}\Phi(\xi,0)\frac{\partial^2}{\partial\xi\partial\eta}\Phi(\xi,0)-\frac{1}{2}\left[\frac{\partial}{\partial\eta}\Phi(\xi,0)\right]^2\right\}. \quad (12)$$

In deep water, the wave profile equation according to (4) is written as

$$\xi_в(\xi)=A\cos\omega\xi+\omega A^2\cos^2\omega\xi-\frac{1}{2}\omega A^2\sin^2\omega\xi. \quad (13)$$

Since the tangent to the wave profile at the top and bottom of the wave is horizontal, i.e.

$$\frac{\partial\xi_в(\xi=0)}{\partial\xi}=0, \quad (14)$$

where $\xi_в(\xi=0)=kr_в$, to determine the constant A, a quadratic equation is used

$$\omega A^2+A-kr_в=0, \quad (15)$$

whose solution has the form

$$A=\frac{1}{2\omega}\left(-1\pm\sqrt{1+4\omega kr_в}\right). \quad (16)$$

The quadratic equation for determining the constant A has two real roots of opposite signs, providing at $\xi = 0$, therefore, at $x = 0$ the same half-height $\xi_в(0) = kr_в$. An unambiguous choice of the value of A is obtained from the condition which means there are no collapsing waves in space (x, z).

$$\frac{d}{d\xi}x(\xi,0) > 0\,, \tag{17}$$

Then

$$\frac{d}{d\xi}x(\xi,0) = 1 - \frac{1}{2}\omega A\cos(\omega\xi), \qquad \left|\frac{1}{2}\omega A\right| < 1 \tag{18}$$

and finally

$$A = \frac{1}{2\omega}\left(-1 + \sqrt{1 + 4\omega kr_в}\right). \tag{19}$$

We obtain the wave profile equation in shallow water by substituting (5) into (12) and retaining terms up to order B^2 inclusive:

$$\xi_в(\xi) = \frac{1}{\omega}\Bigg(\alpha B ch\alpha H\cos\alpha\xi - B^2\alpha^3\frac{ch2\alpha H\cdot\cos 2\alpha\xi}{\omega sh2\alpha H - 2\alpha ch2\alpha H} - $$
$$-\frac{1}{4}B^2\alpha^2\cos 2\alpha\xi + \frac{1}{2\omega}B^2\alpha^3 sh2\alpha H\cos^2\alpha\xi - \frac{1}{4}B^2\alpha^2 sh^2\alpha H + \tag{20}$$
$$+\frac{1}{4}B^2\alpha^2 sh^2\alpha H\cos 2\alpha\xi\Bigg).$$

To determine the constant B we use the expression

$$kr_в = B\left(\frac{\alpha}{\omega}ch\alpha H\right) - B^2\left(\frac{\alpha^3}{\omega}\frac{ch2\alpha H}{\omega sh2\alpha H - 2\alpha ch2\alpha H} + \frac{\alpha^2}{4\omega} - \frac{\alpha^3}{2\omega^2}sh2\alpha H\right). \tag{21}$$

Let us transform it taking into account the dispersion relation (6).
Then

$$\frac{\alpha}{\omega}ch\alpha H = sh\alpha H\,. \tag{22}$$

Further

$$\frac{\alpha^3}{\omega}\frac{ch2\alpha H}{\omega sh2\alpha H-2\alpha ch2\alpha H}=-\alpha\frac{1+th^2\alpha H}{2th\alpha H};\ \frac{\alpha^2}{4\omega}=\frac{\alpha}{4}th\alpha H;$$
$$\frac{\alpha^2}{4\omega}sh^2\alpha H=\frac{\alpha}{4}\frac{th^3\alpha H}{1-th^2\alpha H};\ \frac{\alpha^3}{2\omega^2}sh2\alpha H=\alpha\frac{th^3\alpha H}{1-th^2\alpha H}. \quad (23)$$

Carrying out substitutions and transformations, we get

$$\xi_в(\xi)=Bsh\alpha H\cos\alpha\xi+B^2\alpha\left(\left[\frac{1+th^2\alpha H}{2th\alpha H}-\frac{th\alpha H}{4}+\frac{3}{4}\frac{th^3\alpha H}{1-th^2\alpha H}\right]\times\right.$$
$$\left.\times\cos2\alpha\xi+\frac{1}{4}\frac{th^3\alpha H}{1-th^2\alpha H}\right). \quad (24)$$

From here

$$\xi_в(\xi)=Bsh\alpha H\cos\alpha\xi+B^2\alpha\frac{(2th^4\alpha H-th^2\alpha H+2)\cos2\alpha\xi+th^4\alpha H}{4th\alpha H(1-th^2\alpha H)}. \quad (25)$$

While $\xi = 0$ it turns out

$$kr_в=Bsh\alpha H+B^2\alpha\frac{3th^4\alpha H-th^2\alpha H+2}{4th\alpha H(1-th^2\alpha H)}. \quad (26)$$

Quadratic equation for determining the constant B

$$\frac{\alpha(3th^4\alpha H-th^2\alpha H+2)}{4th\alpha H(1-th^2\alpha H)}B^2+sh\alpha HB-kr_в=0. \quad (27)$$

Equation (27), like (15), has two real roots of opposite signs, ensuring at $\xi = 0$ (i.e. $x = 0$) the same half-height of the wave $\xi_в(0)=kr_в$. The choice of root, as in the case of deep water, is determined by the condition of non-destruction of the wave in the normalized physical space (x, z). This is done if

$$\frac{\partial x(\xi,0)}{\partial\xi}=1-\frac{\alpha B}{2}ch\alpha H\cdot\cos\alpha\xi\geq 0,\quad \frac{\alpha B}{2}ch\alpha H<1. \quad (28)$$

Thus, the constant B is the affirmative root of equation (27).

Let us show that the average level of the excited liquid does not coincide with the unperturbed free surface.

$$\int_{-\frac{\lambda}{2}}^{\frac{\lambda}{2}} \xi_в(x)dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \xi_в(\xi)\frac{dx}{d\xi}d\xi = $$
$$= B^2\alpha\cdot\frac{2\pi}{\alpha}\cdot\frac{1}{4}\left(\frac{th^3\alpha H}{1-th^2\alpha H}-\frac{th\alpha H}{1-th^2\alpha H}\right) = -\frac{B^2\pi}{\alpha}th\alpha H \neq 0. \quad (29)$$

Note also that the wavelength λ as the distance between the vertices of adjacent ridges (or between adjacent antinodes that are in the same oscillation phase) in both spaces (x, z) and (ξ, η) is determined by the formula

$$\lambda = \frac{2\pi}{\alpha} - \frac{1}{2}Bch\,\alpha(\eta+H)\sin\left(\alpha\frac{2\pi}{\alpha}\right) = \frac{2\pi}{\alpha}. \quad (30)$$

Since the space (x, z) is normalized and in this space $\lambda = 2\pi$, $k \equiv 1$, from (30) and (6) it follows

$$\alpha = k = 1, \quad \omega = \frac{1}{thH}. \quad (31)$$

Note that for the case of deep water from (31) we obtain $\omega = 1$, and for shallow water, when $\frac{h}{\lambda} \sim 0(\varepsilon)$, it will be $\omega \sim \frac{\lambda}{2\pi h}$.

Research results. Expression for the wave profile at an arbitrary value of depth, consists of two, linear and quadratic, components relative to the amplitude.

To assess the quality of the obtained solution, the shape of the wave profile was calculated using formulas (6), (20), (27) and (31) for various ratios of wavelength and depth $H = 2\pi\frac{h}{\lambda}$, wave amplitude and wavelength $r = 2\pi\frac{r_в}{\lambda}$.

Results of experimental determination of shape wave profile taken from [10]. The wave profiles corresponding to these data were calculated using the proposed approximate theory, the fifth-order Stokes theory, and the cnoidal theory. In Fig. 1 and 2 show two implementations as examples.

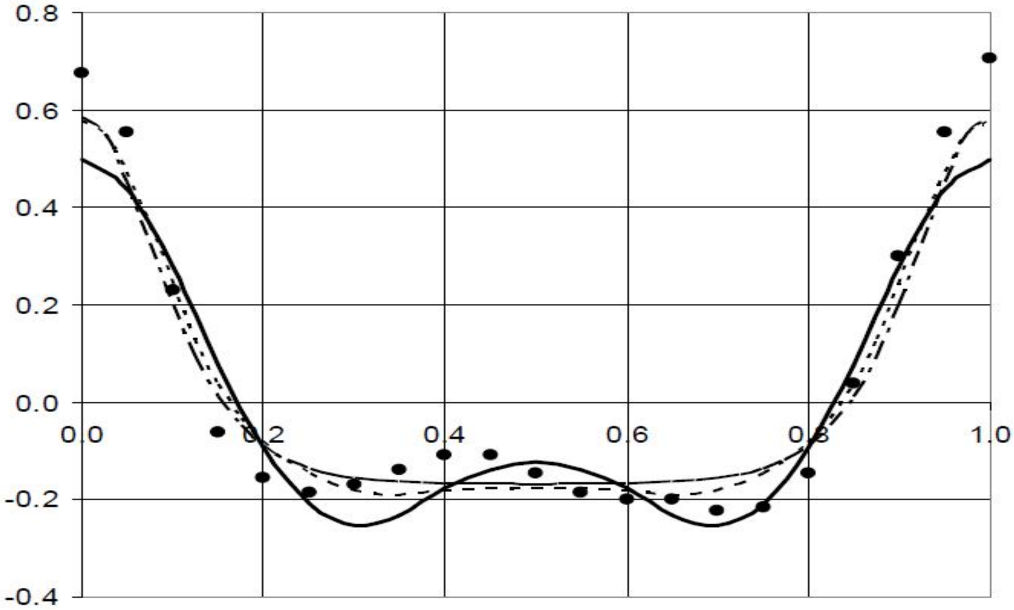

Fig. 1. Relative ordinates of the wave profile at relative depth $H = 2\pi \frac{h}{\lambda} = 0{,}446$

and relative half-height $r = 2\pi \frac{r_в}{\lambda} = 0{,}12$

1 – calculation according to (20);
2 – cnoidal theory;
3 – Stokes theory;
4 – experiment [10];
Ursell number $N_{URS} = 106{,}73$

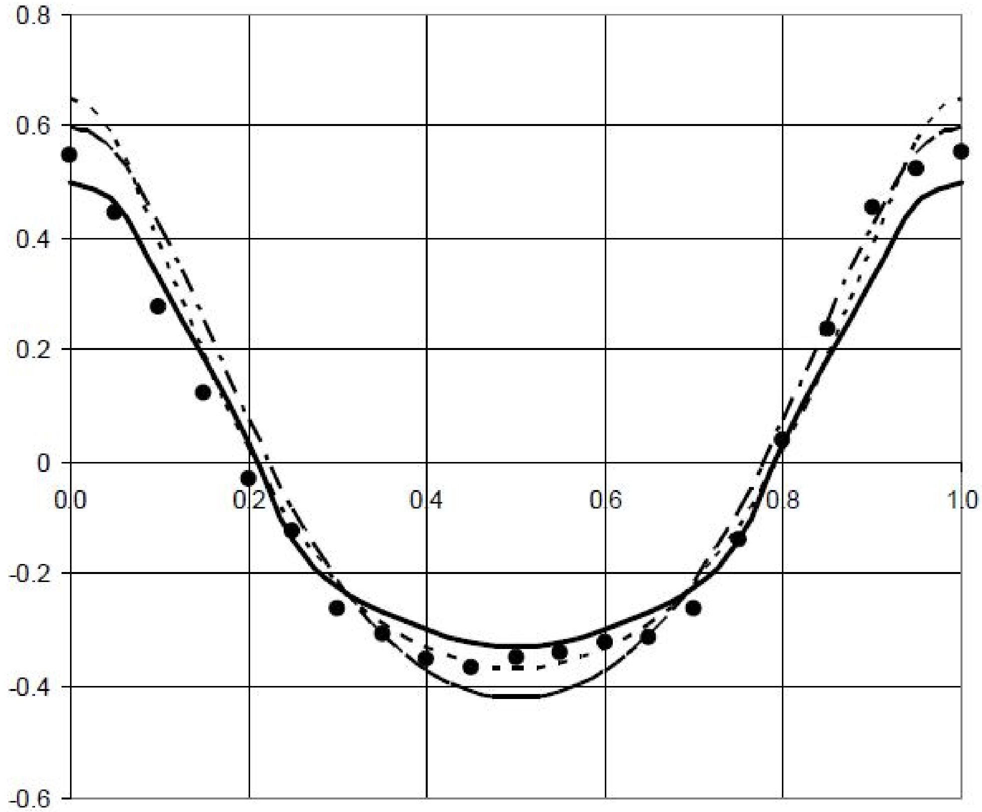

Fig. 2. Relative ordinates of the wave profile at relative depth $H = 2\pi \frac{h}{\lambda} = 0{,}873$

and relative half-height $r = 2\pi \frac{r_в}{\lambda} = 0{,}162$

1 – calculation according to (20);
2 – cnoidal theory;
3 – Stokes theory;
4 – experiment [10];
Ursell number $N_{URS} = 19{,}21$

Conclusions. The results of calculations based on the proposed approximate theory are in quite satisfactory agreement with the experimental data. The wave profiles calculated using the approximate theory, the fifth-order Stokes theory, and the cnoidal theory are qualitatively the same; the quantitative differences between them are of the same order as those given in [2] when comparing the results of calculations of wave profiles using the Stokes theory of the fifth and third orders.

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